Mode-coupling Formation of Complex Modes in a Shielded Nonreciprocal Finline

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Abstract

The use of coupled-mode theory explains qualitatively and quantitatively the kinetic formation of the complex modes, which are explicitly shown to be the result of mode-coupling between a forward wave and a backward wave in a shielded lossless nonreciprocal finline. The unique properties of the complex modes in the nonreciprocal fineline are discussed in detail for the first time. Based on the coupled-mode theory, the amount of coupling between the forward wave and the backward wave can be related to the complex propagation constants of the complex modes, of which the data are obtained by the full-wave spectral-domain approach.

Introduction

In many millimeter-wave and microwave integrated circuits there is a need for nonreciprocal devices such as phase shifters, directional couplers, isolators, and filters. To design these devices successfully, one needs to know the propagation characteristics of the guided-wave structures incorporated in the nonreciprocal devices. In addition to the extensive dominant-mode results published by Geshiro and Itoh[1], it is also important to investigate the higher-order modes excited in conjunction with any discontinuities in the nonreciprocal finline devices[2]. In the nonreciprocal finlines the higher-order modes including the complex modes, to authors' knowledge, have not yet been reported or fully discussed. The excitation of the complex modes, if they exist, needs to be considered when analyzing the waveguide discontinuity problem[3].

The aim of this paper, however, is not limited by presenting the data of the fundamental and higher-order modes including the complex modes for a nonreciprocal finline shown in Fig.1, which consists of the multi-dielectric stratified layers and an additional ferrite substrate magnetized transversely in the x direction. Since the finline under study is nonreciprocal, the modal solutions include the so-called forward wave and the backward wave[1]. The distinction of the forward wave and the backward wave also holds for all the higher-order modes of the nonreciprocal finline. This implies that a deeper physical picture of the formation of the complex modes is possible based on the time-harmonic full-wave modal solutions. Such a distinction between the forward wave and the backward wave allows us to invoke the coupled-mode theory [4] to explain the kinetic mechanism of forming the complex modes in the nonreciprocal finline.

The first step toward the use of the coupled-mode theory is the extensive investigation of a particular nonreciprocal finline under moderate and weak DC magnetic fields (Ho's) applied on the ferrite substrate in the transverse x-direction of Fig.1. Although only two particular case studies are analyzed here, it is believed that the particular examples depict the general dispersion characteristics of the nonreciprocal quasi-planar guided-wave structures. Following the mode charts, obtained by the spectral-domain approach(SDA) with improved accuracy[5], of the nonreciprocal finline under various applied DC magnetic fields, a typical example of contradirectional coupling of a forward propagating wave and a backward propagating wave with various degrees of coupling illustrates the fact that a pair of complex modes exist when the phase constants of the two *contradirectional* propagating modes are *nearly equal.* These contradirctional waves essentially correspond to the *forward* wave and the *backward* wave in the nonreciprocal finline. Then, turning to the mode charts presented earlier and focusing on the regions where the complex modes are formed by a *forward* wave and a *backward*

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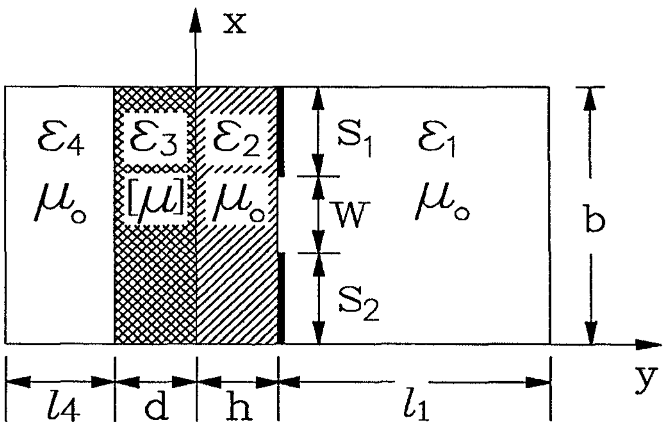

Fig.1 Cross-sectional geometry of a unilateral fineline integrated on the stratified layers containing a ferrite substrate magnitized in x-direction. The structural and material parameters are : l_1=3.556mm, d=1mm, h=1mm, l_4=1.556mm, b=3.556mm, $s_1=s_2$=1.628mm, w=0.3mm, $\epsilon_{1r}=\epsilon_{4r}=1$, $\epsilon_{2r}=\epsilon_{3r}=12.5$, 4πMs=4900G, and Ho=500(30)Oe.

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wave, the coupled-mode theory yields the dispersion characteristics of the *forward* wave and the *backward* wave as well as the complex modes. The values of these propagation constants as predicted by the coupled-mode theory are in excellent aggreement with those obtained by the full-wave SDA. Thus, for the first time, the values of the complex modes obtained by the full-wave approach in a nonreciprocal finline can be directly related to the amount of coupling between a *forward* wave and a *backward* wave.

The Nonreciprocal Finline Model and The Method of Analysis

Fig.1 shows the nonreciprocal finline model investigated in this paper. The finline consists of the multi-dielectric layers and a ferrite substrate magnetized in the x-direction by adjusting the DC magnetic field Ho across the ferrite substrate. The finline cross-section is subdivided into various regions designated by the corresponding material parameters. All the data presented herein have the same structural parameters and material parameters, which are indicated in Fig.1, except that only the applied DC magnetic fields(Ho's) are different. One is for the moderate amount of applied DC magnetic field on the ferrite substrate, and the other is relatively weak. By doing so, the general dispersion characteristics of the nonreciprocal finline can be obtained and compared by the two case studies covering fairly broad range of magnetization.

The theoretical data obtained by the spectral-domain approach are validated first by having very good agreement with those reported in Fig. 2 of [1]. The number of the spectral terms used in obtaining the full-wave solutions for the dispersion characteristics of the nonreciprocal finline is 2000 and the set of basis functions employed here has degree of three. Thus a 12 by 12 characteristics matrix equation is derived for solving the propagation constant accurately[5]. The time-harmonic $e^{j\omega t}$ factor and the propagating $e^{-j\gamma z}$ factor are assumed, where $\gamma=\beta-j\alpha$. When Ho and Ms are both positive, one obtains the solutions for either the positive-going propagating (forward) wave with $\beta > 0$ and $\alpha = 0$ or the negative-going propagating (backward) wave with $\beta < 0$ and $\alpha = 0$. When Ho and Ms are both negative, all the signs of the above-mentioned modal solutions are reversed. Note that the convention for the counterpropagating modes holds for both dominant and higher-order modes.

Dispersion Characteristics of a Nonreciprocal Finline

Figs.2(a) and (b) plot the dispersion characteristics of the nonreciprocal finline of Fig.1 with moderate and weak applied DC magnetic fields on the ferrite substrate, respectively. Following the discussions mentioned in the previous section, one only needs to find the modal solutions when both *Ho* and Ms are positive. Under this condition, the modal solutions for the normalized propagation constants can be either designated as F1, F2, ..., etc., of which the leading letter F stands for the *forward* positive-going propagating wave, or B1, B2, ..., etc., of which the leading letter B means the *backward* negative-going propagating wave.

In both plots, clearly, the complex modes exist and occur when a pair of a forward wave and a backward wave start to become evanescent modes (below cut-off). In the particular case studies, however, every pair of the forward wave and the backward wave can never become evanescent modes. Instead, they form a pair of the complex modes at a frequency point where the group velocities are zero.

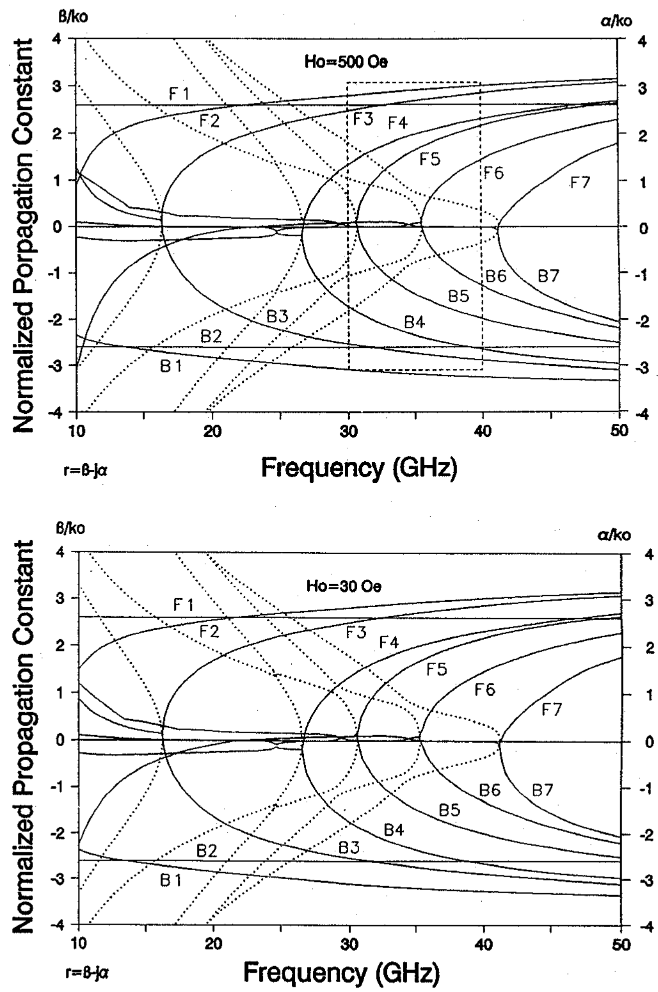

Fig.2 Dispersion characteristics (mode charts) of Fig.1. The solid lines and dot lines are for the real and imaginary parts of the normalized propagation constants, repectively. (a) Ho=500Oe, (b) Ho=30Oe.

Since very similar dispersion characteristics are obtained in Fig.2(a) and Fig.2(b), it is plausible to consider the above-mentioned findings hold for general shielded nonreciprocal quasi-planar guided-wave structures.

Coupled-Mode Theory and The Complex Modes

Following the same terminologies used in [4], let the propagation constants of the *hypothetical* uncoupled modes be β_p and β_q, respectively, and the coupling factor of these two modes is K. The propagation constants of the resultant coupled modes β_1 and β_2 on the continuously coupled condition can be shown to be

$$\beta_1 = (\beta_p+\beta_q)/2 + \sqrt{[(\beta_p-\beta_q)/2]^2 \pm K^2} \qquad (1)$$

$$\beta_2 = (\beta_p+\beta_q)/2 - \sqrt{[(\beta_p-\beta_q)/2]^2 \pm K^2} \qquad (2)$$

When β_p and β_q are in the same direction, the '+' sign applies. When β_p and β_q are contradirectional, the '-' sign applies. Obviously, β_p and β_q need to be contradirectional to become the complex modes. Fig.3 illustrates this observation for the two contradirectional modes β_p and β_q with various values of the coupling factor K. The resultant *coupled-mode solutions* , β_1 and β_2, do establish regions where the complex modes exist. The higher the value of K is, the wider of the complex modes region is established and the bigger the absolute value of the imaginary(attenuated) part of the corresponding complex propagation constants.

Note that, in Fig.3, β_p and β_q are the forward wave and the backward wave, respectively. Significant coupling among the *two hypothetical uncoupled modes,* β_p and β_q , occurs when the propagation constants of the two modes before coupling are *nearly equal.* They form the complex modes, which are in complex conjugate pairs. Of more importance is the fact that the plots of

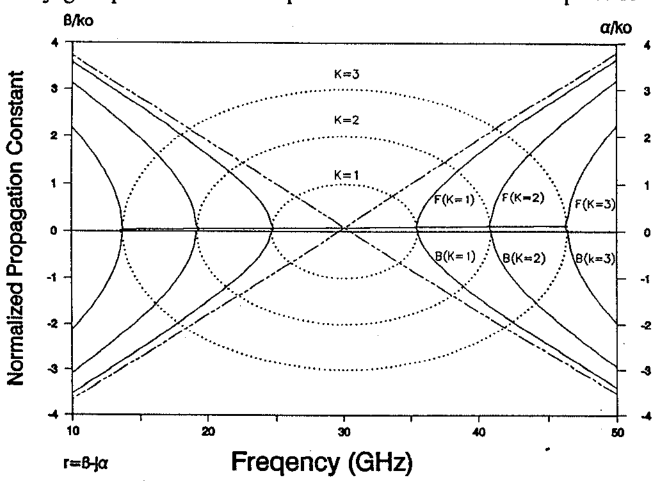

Fig.3 The complex modes formed by two contradirectional modes as predicted by the coupled-mode theory. The real parts of normalized propagation constant are plotted with solid lines. The imaginary parts are plotted with dot lines.

the resultant coupled modes, β_1 and β_2, bear very close resemblance to those of Figs.2(a) and (b) in terms of the physical appearance of the modal solutions for the dispersion characteristics of the nonreciprocal finline. This can be attributed to the following three observations among these plots.

First, in the complex modes region, they all have complex propagation constants in complex-conjugate pairs. Second, all the complex modes, regardless of whether they are obtained by the full-wave SDA analyses or by the couple-mode theory, start at the frequency points where the group velocities are zero. Third, the complex modes can be established by a pair of a forward propagating wave and a backward propagating wave. These observations encourage us to investigate the full-wave modal solutions of Figs.2(a) and (b) quantitatively by the coupled-mode theory in the next section.

Application of The Coupled-Mode Theory To The Full-Wave SDA Modal Solutions of A Nonreciprocal Finline

To finalize the legitimacy of applying the coupled-mode theory to explain the formation of the complex modes in the nonreciprocal finline, one needs the quantitative evidence in addition to the qualitative discussions presented in the previous section. Since an actual shielded waveguide has infinite number of modes, these modes are likely to couple each other. Thus we frame only part of the region for modes F_6 and B_6 in Fig.2(a). In this region one may consider that these two modes interact more strongly than other higher or lower-order modes including the complex modes. Therefore the simple coupled-mode theory may apply.

Inside the framed region of Fig.2(a), there are three types of modes, namely, the forward propagating wave, the backward propagating wave, and the complex modes. Using the data inside the framed region, we can obtain the approximate solutions for β_p and β_q. This is done first by solving β_p and β_q in terms of β_1 and β_2 in (1) and (2). Then plugging into the new expressions using the data inside the framed region of Fig.2(a) with a proper choice for the value of K, one obtains the curved solutions for β_p and β_q. Finally we linearize those curved solutions by the application of the least-square error algorithm and the results are the two straight dash-dot lines illustrated in Fig.4.

In Fig.4, there are another two sets of data. The full-wave SDA data of Fig.2(a) are in solid lines. The computed β_1 and β_2 from the data of β_p and β_q using the coupled-mode theory are in dot lines. The data represented by dot lines agree excellently with those by the solid lines in the entire spectrum of Fig.4.

In summary, one may model the kinetic formation of the complex modes in a nonreciprocal finline to such an extent that the full-wave modal solutions in the neighborhood of the complex modes can be accurately predicted by applying the coupled-mode theory on the two *hypothetical* uncoupled modes. For the particular case study, these two hypothetical uncoupled modes can be found numerically from the full-wave modal solutions.

Conclusion

The complex modes pose several unique properties to the full-wave SDA modal solutions of a shielded lossless nonreciprocal finline. There are no evanescent modes found in the particular case studies of the nonreciprocal finline. Instead, the complex modes replace the regions that would have the evanescent modes. These complex modes occur in complex-conjugate pairs, which are formed by a pair of a forward wave and a backward wave. The occurrence of the complex modes starts at a frequency point where the group velocity is zero.

The kinetic formation of the complex modes in the nonreciprocal finline can be described by the coupled-mode theory both qualitatively and quantitatively. The coupled-mode theory explains why no evanescent modes are found in the full-wave modal solutions for the particular finline. It manifests the fact that the complex modes can be establishd by two contradirectional modes when the propagation constants of those two modes are nearly equal. Both the full-wave dispersion data and the predicted modal solutions based on the coupled-mode theory are in excellent agreement in the neighborhood of the complex modes region, in which there are the propagating forward wave, the propagating backward wave, and the complex waves. Since the coupled-mode theory model these modes accurately, the myth of the complex modes in the shielded lossless nonreciprocal finline is uncovered.

Acknowledgement

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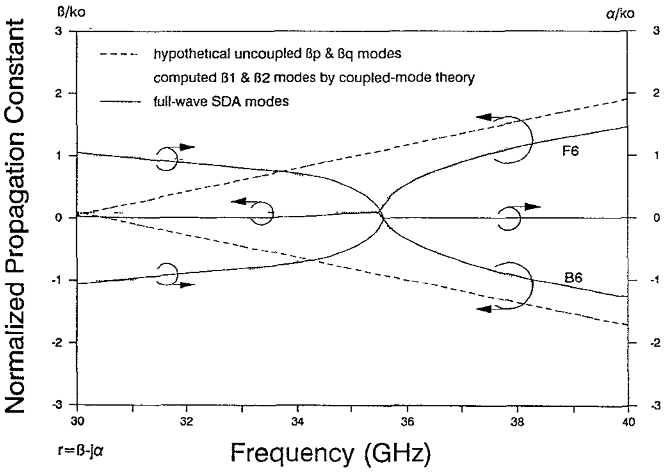

Fig.4 The comparison of the finline dispersion characteristics using the data predicted by the coupled-mode theory and those obtained by the full-wave SDA. The coupled-mode theory models the forward wave, the backward wave, and the complex modes accurately.

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